THE WORK OF YVES MEYER

In his famous 1960 essay "The unreasonable effectiveness of mathematics in the natural sciences", Eugene Wigner noted the uncanny ability of mathematical notions and discoveries, that were often pursued for no other reason than their intrinsic structure and beauty, to become highly relevant in describing the physical world. The work of the 2017 Abel laureate, Yves Meyer, exemplifies this ability of pure mathematics to cross over into practical real-world applications.

The *Fibonacci numbers* $1, 1, 2, 3, 5, 8, \dots$ are a simple example of an object from pure mathematics that appears in surprising ways in nature. The ratios $1/1, 2/1, 3/2, 5/3, 8/5, \dots$ of consecutive Fibonacci numbers converge extremely rapidly to the famous *golden ratio* $\phi = \frac{1+\sqrt{5}}{2} = 1.61803\dots$. This number is special in many ways. The powers $\phi, \phi^2, \phi^3, \dots$ of the golden ratio lie unexpectedly close to integers: for instance, $\phi^{11} = 199.005\dots$ is unusually close to 199. Meyer's early work focused on a class of numbers (including the golden ratio) with this property, known as *Pisot numbers*. He discovered that one could use these Pisot numbers to create sets of points (now known as *Meyer sets*) in a line, a plane, or in higher dimensions that behaved almost, but not quite, like the periodic sets of points one sees in the integers on the real line, or the grid points of a Cartesian plane. A simple example of such a set would be the collection of numbers, such as $\phi + \phi^3 + \phi^4$, that can be formed by adding together distinct powers of the golden ratio ϕ. Such sets of points are not perfectly periodic, but have a property known as *almost periodicity*: any pattern that one sees in the set will recur infinitely often, albeit not at perfectly regular intervals. Meyer was motivated to construct these sets to answer purely theoretical questions in the study of Fourier series (superpositions of sinusoidal waves); but a decade after Meyer's work, it was discovered that Meyer sets could be used to help explain the physical properties of *quasicrystals* – arrangements of molecules that are not periodic in the way that genuine crystals are, but still behave like crystals in many key ways, such as in their diffraction pattern. (The physical discovery of quasicrystals by Dan Schechtman was recognized by the Nobel Prize in Chemistry in 2011.)

One of Yves Meyer's early research interests was the study of *singular integral operators* – certain integrals arising in such fields as Fourier analysis, complex analysis, and partial differential equations that are only finite due to delicate cancellations and oscillations in the expressions being integrated. One of the basic tools used to analyze these integrals was the *Calderón reproducing formula*, that allowed one to express an arbitrary function in space as a combination of simpler objects that were localized in space while also being smooth and somewhat oscillatory. Meanwhile, motivated by applications in geophysics, Morlet and Grossmann were also experimenting with analyzing time series data (such as seismic data) in both time and frequency simultaneously, by measuring how these data correlated with windowed cosine waves, where the width of the window varied inversely with the frequency of the wave. Meyer realized that the two transforms were essentially identical to each other; this insight then led to the development by Meyer and others of the *wavelet transform* that allowed one to efficiently and easily decompose any signal into localized oscillatory objects now known as *wavelets*. This transform captured many of the beneficial features of the more classical Fourier transform (in particular, the ability to separate out the fine-scale aspects of the data from coarse-scale aspects), while suffering fewer of the drawbacks (in particular, information about spatial features of the data, such as edges or spikes, were much more visible using the wavelet transform than with the Fourier transform). This began the "wavelet revolution" of signal processing in the late 1980s and early 1990s, with the wavelet transform now being routinely used in many basic signal processing tasks such as compression (e.g. in the JPEG2000 image compression format) and denoising, as well as more modern applications such as compressed sensing (reconstructing a signal using an unusually small number of measurements).

Meyer's intuition on the interplay between low and high frequency components of functions led to many important theoretical advances in the fields of harmonic analysis and partial differential equations, ranging from the solution of key open problems such as the boundedness of the Cauchy integral operator on Lipschitz curves (solved by Coifman, McIntosh, and Meyer), to the development of new tools such as *compensated compactness*, *paraproducts*, and *paradifferential calculus* that are now indispensable in the understanding of nonlinear effects in partial differential equations, particularly for equations that govern such oscillatory behavior as the motion of waves in a medium. For instance, the important but still poorly understood phenomenon of *turbulence* in fluids, in which the velocity field becomes increasingly oscillatory and fine-scaled in behavior, can be at least partially explained by considering how various wavelet coefficients of the fluid interact with each other, and using the technical tool of paraproducts to measure the strength of such interactions; this has proven to be influential both in the theoretical analysis of the equations of motion of these fluids, as well as in the numerical methods used to simulate these fluids. Meyer's work and insight has not only advanced the pure and applied sides of mathematical analysis, it has also brought them together in a tightly interconnected fashion.