Abel Prize 2008

Thompson and Tits, Abel and Lie

The Abel Commitee writes in the citation that the work of John G. Thompson and Jacques Tits has been complementary, one in the direction of finite groups, the other in the direction of linear groups. This dichotomy has its analogue in the relation between the two most important Norwegian mathematicians, Niels Henrik Abel and Sophus Lie.

The most important groups are finite groups, arising for example in the study of permutations, and linear groups, which are made up of symmetries that preserve an underlying geometry. The work of the two laureates has been complementary: John Thompson concentrated on finite groups, while Jacques Tits worked predominantly with linear groups.

In this way the Abel Committee argues for their choice of a joint Abel prize to John G. Thompson and Jacques Tits, two mathematicians who each in his own has been central in the development of group theory, but two who have never published a joint paper.

The Norwegian history of mathematics is also dominated by two giants, Niels Henrik Abel and Sophus Lie. For obvious reasons, neither did they publish any joint paper, as Abel died 13 years before Lie was born. But as is the case of Thompson and Tits, Abel and Lie are central names in the history of group theory. An evidence of this fact is that important classes of groups carry their names, namely 'abelian groups' and 'Lie groups'. It is common practise in mathematics to name objects and results after the founder, but to become an adjective, like Abel, is rather rare.

Niels Henrik Abel had his time long before the notion of a group became folklore. But in his work one can find the germ of the theory of finite groups and the definition of an abelian group can be read between the lines. His result about the insolubility of a general quintic equation by radicals is based on the insolubility of what was later known as 'Galois groups', again indirectly defined by Abel.

Working over more than three decades after the first occurrence of the term 'group' in the mathematical testament of Galois, Sophus Lie was more aware of the concept of a group. Lie's work concerns continuous groups. Lie was inspired by Abel's idea of understanding equations by studying permutations of the solutions. He transferred the philosophy to solutions of differential equations. This led him from finite groups of permutations to infinite, continuous transformation groups.

Once again we quote the Abel Committee in their citation. But this time we substitute Thompson and Tits by Abel and Lie. The argument is still valid.

The most important groups are finite groups, arising for example in the study of permutations, and linear groups, which are made up of symmetries that preserve an underlying geometry. The work of the two giants of Norwegian mathematics has been complementary: Niels Henrik Abel concentrated on finite groups, while Sophus Lie worked predominantly with linear groups.