Abel Prize 2008

Why Jacques Tits is awarded the Abel Prize for 2008

The citation of the Abel Committee gives the official reason for awarding the Abel Prize to Jacques Tits. Here are some more details for non-experts.

The Abel Committee writes in the citation for the Abel Prize for 2008 about ***Jacques Tits:***

> *Tits created a new and highly influential vision of groups as geometric objects. He introduced what is now known as a Tits building, which encodes in geometric terms the algebraic structure of linear groups.*

Just like Thompson, Tits is working in group theory, but there is a difference in their area of interest. Whereas Thompson mainly works on finite groups, Tits' interests are in *linear groups*, which are likely to be infinite.
However, the two concepts are not so different, satisfying the same set of axioms. Two related symmetry groups illustrate the differences between finite and infinite groups. In a given circle we inscribe a regular n-gon, with all edges of the same length. A regular n-gon has 2n symmetries, n rotations with one side up, and n rotations with the other side up. If we let the number of edges approach infinity, the vertices of the polygon is smoothened, turning the polygon into a circle. The same happens to the symmetry group, changing from a finite group in the non-smooth case of the n-gon into a continuous group in the smooth case of the circle. This illustrates the nature of the linear groups.
Tits created in his work in the sixties a geometrical framework to study linear groups. In a fabulous theoretical system, borrowing names to all concepts from architecture, Tits could give a geometrical description of purely algebraic structures. Tits presents *buildings, apartments and galleries*, and the names of the components of the construction help the reader to create a fruitful intuition in difficult algebraic questions.

The Tits-architecture was not only a spectacular construction of theoretical interest. The Committee refers to several applications of the construction:

> *The theory of buildings is a central unifying principle with an amazing range of applications, for example to the classification of algebraic and Lie groups as well as finite simple groups, to Kac-Moody groups (used by theoretical physicists), to combinatorial geometry (used in computer science), and to the study of rigidity phenomena in negatively curved spaces.*

Another field, mentioned by the Committee in the citation, links the two laureates:

> *Tits's geometric approach was essential in the study and realisation of the sporadic groups, including the Monster.*

One of the sporadic groups is often linked to Tits; the so-called *Janko-Hall group*. Tits described this group as the automorphism group of a graph with 100 vertices and 1800 edges. It has order 604 800 and Tits often makes a joke of this number: *The order 604 800 of the Janko-Hall group equals the number of seconds in a week.* Tits also has his name linked to yet another branch of group theory, and we quote the Committee:

> *[Tits] also established the celebrated "Tits alternative": every finitely generated linear group is either virtually solvable or contains a copy of the free group on two generators. This result has inspired numerous variations and applications.*

A *virtually soluble group* is a group containing a

soluble group as a subgroup of finite index. A *free group* on two generators is generated by the two generators, and there are no relations between the generators, nor relations between any exponents of the generators. Hence, if a finitely generated linear group is such that we cannot find two totally independent generators, then at least a subgroup of the group, which is almost as big as the group itself, is soluble. The 'Tits alternative' is yet another deep and powerful result due to Tits that enlightens the theory of groups.