The Abel Prize Winner 2013 Pierre Deligne

Popular version by Arne B. Sletsjøe

Riemann Hilbert Correspondance

At the International Congress of Mathematicians (ICM) in 1900, David Hilbert put forth a celebrated list of 23 problems. The twenty-first problem was phrased like this: "In the theory of linear differential equations with one independent variable z, I wish to indicate an important problem, one which very likely Riemann himself may have had in mind. This problem is as follows : To show that there always exists a linear differential equation of the Fuchsian class, with given singular points and monodromic group. ....."

A differential equation is an equation that gives a relation between a function and how the function changes. Differential equations have been studied since the seventeenth century and is an important mathematical tool for understanding phenomena in the nature.

One such phenomenon is a whirlpool. Using basic laws of nature and knowledge of the behaviour of liquids, we can suggest a differential equation that describes what happens in the bathtub when we remove the stopper. The solution of the equation describes a whirlpool motion, close to what is observed in the liquid.

The difference between the mathematical model, expressed by the differential equation, and the real motion of the liquid, increases as we approaches the centre of the whirlpool. In the

bathtub there is no water at all in the centre of the whirlpool, it has already been drained out. In the model, however, the speed of the circular motion of the water will increase as we approach the centre. In the centre, the model will collapse. The model has a **singularity**in this point.
Such singularities have an interesting effect on the solutions of the differential equation. A mathematical phenomenon called **monodromy** may occur. The word monodromy is of Greek origin and means running round simply.
Now, suppose we have found a solution of the differential equation. The value of the function will vary continuously along arbitrary curves. When we return to the starting point, the value of the function will be the same as when we started. I.e., as long as we do not run round a path encircling the singularity. In that case the value might change. This is what is called monodromy. It is a bit like motions in a spiral staircase. As long as the full loop does not encircle the centre of the staircase, we remain at the same level, but encircling the centre will bring us either upwards or downwards.

A differential equation can have different numbers and types of singularities, and also different types of monodromy. Hilbert was well aware of this fact. What he wondered about was the opposite question: Given the monodromy and the singularities, can we always find a differential equation?

The problem has elicited many answers during the 20th century. At the same time the problem has been generalised in many directions. The complete solution of the most general version of the problem is due to Pierre Deligne.