Abel Prize 2009

Popularized description of Riemannian and symplectic geometry

The word geometry has its origin in ancient Greek; γεωμετρία; geo = earth, metria = measure. It is the part of mathematics that deals with questions relating to size, shape, relative positions of figures and with properties of space. It is one of the oldest sciences we have. The basic objects in geometry are points, lines and planes. As early as in the 3rd century BC Euclid described plane geometry as we experience it on a sheet of paper. We have an intuitive understanding of what "length" and "area" are. If we consider a more curved surface, like a geographical region or the surface of our body, the notions of length and area are no longer that obvious.

Consider the following problem. Imagine that you have a small farm in the western part of Norway, and you want to grow grass on your fields. Looking at the map, your property looks like a nice rectangle, and computing the area in order to calculate the amount of seed needed for the grass production looks like an easy task.

The problem is that your field is far from flat. It looks more like the skin of a puppy or the ocean just after a big storm. A small square on the map corresponds to a (much) bigger area of the field. The relation between the two sizes is measured by what is called a ***2-form*** on the surface. Once we know this 2-form in each point of the surface we are able to compute the actually area of our field. A surface with a 2-form is called a ***symplectic surface*** and the theory for such objects is known as ***symplectic geometry.***

Consider another problem. During the summer months your sheep are running free up in the mountains. As the responsible caretaker, you have to walk around in the mountains to look after the animals. Even if this mountain area is rather flat it is not obvious that the shortest way between two points is the straight line. In some places the ground is dry, in other places there are solid rocks and in some places you walk in a wet marsh. It is much harder to walk in the marsh than on dry ground. Now we examine every point very carefully. In every single point we attach a value to each direction. The value measures how hard it is to walk in the given direction at that particular point. In the marsh the values are high compared to the dry ground. The various values in a point reflect variations in the ground in different directions. Now we track a route. The time or energy spent to walk through the route is found by "adding" up the values in the appropriate directions at every point of the route. The described function is a ***metric*** on the surface, and a surface equipped with such a metric is called a ***metric surface***. The theory of metric surfaces and other manifolds is known as ***Riemannian geometry***.