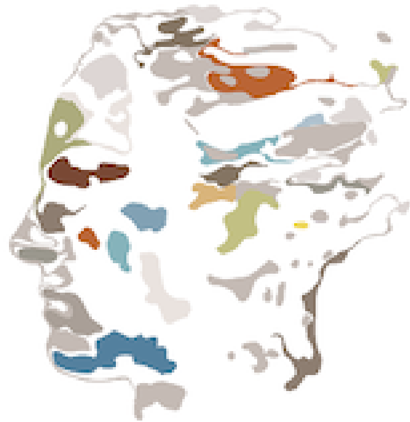
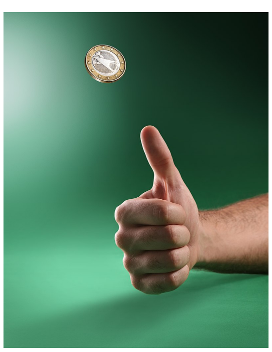


Michel Talagrand
Abel Prize Laureate 2024

Stochastic processes

”Much of Talagrand´s work concerns the geometry of stochastic processes.” (quote from the citation of The Abel Committee)

If you toss a coin once it can either show head or tail. For computational reasons we rename head as 1 and tail as 0. The two outcomes have equal probability, i.e. one half each. The expected value is the sum of products of outcome times probability. For the tossing of a coin the expected value is $\frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 0 = \frac{1}{2}$, even though this value is never achieved.



Source: Wikipedia/ICMA Photos

If we in stead use 10 coins, or toss a coin 10 times, we can compute an outcome by counting the number of heads and divide by 10, the number of tosses. The expected value is still $\frac{1}{2}$, but now this value is achievable if the experiment results in 5 heads and 5 tails. Out of the 1024 possible outcomes of the 10 tosses, 252 or approximately 25% will contain 5 heads and 5 tails. If we expand the acceptable outcome range to include also 4 og 6 heads, the success rate jumps up to approximately 66%. The mean value of heads for 10 coins is 5, so 4 or 6 can be thought of as 20% off the mean. For 20 coins a deviation less than 20% off the mean correspond to 8, 9, 10, 11 or 12 heads out of 20. A computation shows that in this case these number of heads takes up around 75% of all possible outcomes. If we increase the number of coins even more, this percentage will also increase and approach 1, as we increase the number of tosses.

We can generalise this example by replacing the tossing of a coin by a stocastic variable. A stochastic or random variable is a variable whose values randomly change according to some probability distribution law. The set of values the stochastic variable can assume is called the sample space. The stochastic variable which models tossing of coins has a sample space of two elements; {Head, Tail}, with equal probability. Notice also that the outcome of one tossing is independent of the outcome of all the others.

In a more general setting the sample space is likely to be much more complicated than the two outcomes of a coin tossing. Throwing a dice will give a sample space of 6 outcomes, while the price of a share on the stock exchange can vary over a rather large range. But unlike the coin or the dice, the price of the share measured day by day is not an independent variable. The price the next day will not differ very much from the day before. The stochastic variables are corelated.

Talagrand has been interested in this problem, and provided important results for the bound of a large collection of correlated random variables.