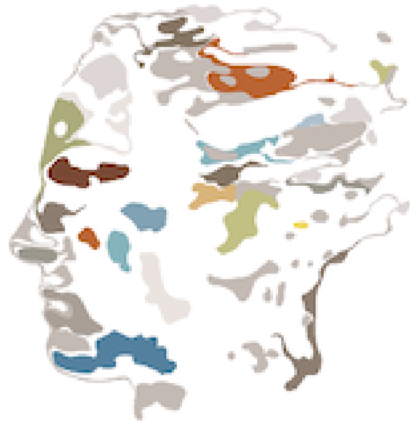

Michel Talagrand
Abel Prize Laureate 2024

Michel Talagrand

Source: Hong Kong Laureate Forum

Michel Talagrand, Centre national de la recherche scientifique, Paris, France is awarded the Abel Prize for 2024

”for his groundbreaking contributions to probability theory and functional analysis, with outstanding applications in mathematical physics and statistics.”

Deviation from the expected

The Abel Committee says in the citation: ” The development of probability theory was originally motivated by problems that arose in the context of gambling or assessing risk. It has now become apparent that a thorough understanding of random phenomena is essential in today´s world. For example, random algorithms underpin our weather forecast and large language models. In our quest for miniaturisation, we must consider effects like the random nature of impurities in crystals, thermal fluctuations in electric circuits, and decoherence of quantum computers. Talagrand has tackled many fundamental questions arising at the core of our mathematical description of such phenomena.”

In 2007, S. R. Srinivasa Varadhan was awarded the Abel Prize, also for his contributions to probability theory. The connection between the two laureates can be illustrated by the law of large numbers. The law of large numbers was first proved by the Swiss mathematician Jacob Bernoulli in his treatise Ars Conjectandi, published in 1713, 8 years after his death.

Bernoulli studies an event that occurs with a certain probability, but the probability is unfortunately unknown to him. His aim is therefore to estimate the probability as the fraction of the number of times the event occurs by the number of times the experiment is repeated.

Bernoulli´s task is to estimate the proportion of white balls in an urn that contains an unknown number of white and black balls. His strategy is to draw a sequence of n balls from the urn, putting the ball back after each draw, and estimate the unknown proportion of white balls in the urn by the proportion of the balls drawn that are white. Bernouilli shows, by choosing n large enough, that he can obtain any desired accuracy and reliability for the estimate. This is precisely the weak version of the law of large numbers.

The law tells us that the probability of the difference between the mean and the expectation being less than some chosen value, is as close to 1 as we would like, provided we repeat the experiment sufficiently many times. If we have found twice as many white as black balls after a large number of trials, we have increasing evidence for the same proportion inside the urn.

Among the important contributions by Talagrand we find a variety of results gathered under the umbrella ”Talagrand´s inequalities”. The deviation from the expected value in the law of large numbers is an archetype of an inequality in this context; give an upper bound for the probability of a given deviation as a function of the probability of the event and the number of repetitions of the event.

J. Bernoulli (1654-1705)


S. R. Varadhan (1940-)

Source: Wikipedia/Abelprisen

Former Abel Prize laureate Varadhan studied the tail of the probability distribution, i.e. the occurrence of events outside of the range of the inequality, popularly stated as ”the unexpected is also expected”. Varadhan´s approach can be illustrated by one of the headaches of the insurance company; how can we be prepared for the unexpected, like a 100-year flood or an earthquake in a geologically stable area? Talagrand´s approach is to give a range where the actual probability model is likely to be valid, e.g. how precise is the weather forecast or to what extent can we know that the AI robot is not hallucinating?

Transportation costs between measures

Another important contribution mentioned by the Abel Committee is ”a useful inequality bounding the quadratic transportation cost distance between a probability measure and a Gaussian distribution by their relative entropy.” To a non-expert this might seem more like a collection of strange word, rather than a reasonable sentence.

The content of that phrase can be illustrated by the following example: The portfolio of the NOK 17-trillion Norwegian Government Pension Fund Global is spread over a large number of investments. Every day the analysts trade shares and bonds in a large scale, trying to optimize the return of the fund. Every transaction has an individual cost, and an important challenge for the analysts is to decide how to ”restructure” the portfolio to the lowest possible total cost. If we say that the total amount of money in the fund is 1, we can consider the portfolio as a probability distribution over the set of banks, funds, companies and other investment objects. The profile of the portfolio two consecutive days describes two different probability measures on the same set, and the strategy for how to move money between the investment objects has a price, given by the cost function. Thus, each moving strategy has an individual cost.

We can ”apply” Talagrand´s inequality for the bounding of transportation cost to this illustrating example. The result would then say that the optimal solution to minimizing the transportation cost for the rearrangement of the portfolio is essentially bounded by a numerical value for the dependence of the two portfolio distributions. This numerical value is what is called the relative entropy. It measures essentially the difference between two profiles of the portfolio. If the profiles of the portfolio are the same, then there are no transportation costs between them. If the variation is significant, the transportation cost will increase. In both cases, the transportation costs are essentially controlled by the relative entropy of the two profiles, as pointed out by Talagrand.

Bounds of the free energy for spin glasses

Another quote from the citation, referring to the Parisi formula for the Sherrington-Kirkpatrick model for the free energy of spin glass: ”... this formula is an upper bound for the free energy. In a groundbreaking article, Talagrand proved the complementary lower bound, hence completing the proof of the Parisi formula."

Spin glasses are alloys formed by noble metals in which a small amount of iron is dissolved. The iron molecules are too distant to really affect each other, and it is only when influenced by an external magnetic field they close the ranks and becomes a nice ferromagnet. Spin glasses differ from their ferromagnetic relatives by the fact that when the external field is removed, the magnetization of the spin glasses will decrease rapidly. The phenomenon is described by considerations around the free energy of the system. The Parisi formula for the Sherrington-Kirkpatrick model refers to a long-standing conjecture about the bounds of the free energy.

Mathematically we can consider the spin of each iron molecule as a random variable. The macroscopic behavior of the spin glass is therefore related to the asymptotic values of a large number of weakly correlated random variables. This indicates the link to the interests of this year´s Abel Laureate. Talagrand´s proof of the Parisi formula is a manifestation of how mathematical theory can contribute to increased knowledge in subjects outside mathematics.